Dilaton coupling revisited

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Abstract

We reinvestigate the dilaton coupling in the string theory, which comes from a wrapped membrane. The ghost number anomaly associated with the string world-sheet diffeomorphism is shown to induce the dilaton coupling.

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1 Introduction

Supermembrane in eleven dimensions [1] is an important object in M-theory and the relation with superstring is well known. In fact, through the double dimensional reduction [2] the wrapped supermembrane on $\mathbb{R}^{10} \times S^1$ is reduced to type IIA superstring on $\mathbb{R}^{10}$. Furthermore, it was explicitly shown that the (p, q)-string [3, 4] in type IIB theory is obtained from the wrapped supermembrane on the T^2-compactified target space through the double dimensional reduction [5, 6, 7]. This also indicates that the duality in type IIB theory is naturally understood in M-theory [3, 8].

Recently, the coupling of the string worldsheet Euler character χ to the dilaton ϕ has been studied in M-theory [9]. It was presented that the $\chi\phi$-term in type IIA theory arises from the measure of the membrane partition function. This may also indicate that the membrane explains the properties of string theory. In this paper we reinvestigate the membrane origin of the dilaton term by the Fujikawa method [10, 11] and we will see that the path integral measure leads to the coupling of the dilaton ϕ to the Ricci scalar $R^{(2)}$ of the string worldsheet metric γ

$$\int_\Sigma d^2\sigma \sqrt{-\gamma}\, R^{(2)} \phi \,. \tag{1.1}$$

This paper is organized as follows. In the next section, we deduce the string action from the wrapped supermembrane by the double dimensional reduction. In section 3, we seek for the dilation coupling with the partition function of the reduced string action. The final section is devoted to discussion.

2 Double dimensional reduction

The action of a supermembrane coupled to an eleven-dimensional supergravity background is given by [1]

$$S = T \int d^3\sigma \left[-\frac{1}{2}\sqrt{-\hat{\gamma}}\,\hat{\gamma}^{\hat{i}\hat{j}}\,\hat{\Pi}_{\hat{i}}{}^{A}\hat{\Pi}_{\hat{j}}{}^{B}\,\eta_{AB} + \frac{1}{2}\sqrt{-\hat{\gamma}} - \frac{1}{3!}\,\epsilon^{\hat{i}\hat{j}\hat{k}}\,\partial_{\hat{i}}Z^{\hat{M}}\partial_{\hat{j}}Z^{\hat{N}}\partial_{\hat{k}}Z^{\hat{P}}\hat{C}_{\hat{P}\hat{N}\hat{M}} \right], \tag{2.1}$$

where

$$\hat{\Pi}_{\hat{i}}{}^{\hat{A}} = \partial_{\hat{i}} Z^{\hat{M}} \hat{E}_{\hat{M}}^{\hat{A}} \,, \tag{2.2}$$

T is the tension of the supermembrane,[1] $\hat{C}_{\hat{M}\hat{N}\hat{P}}(Z)$ is the super three-form, $\hat{\gamma}_{\hat{i}\hat{j}}$ $(\hat{i}, \hat{j} = 0, 1, 2)$ is the worldvolume metric, $\hat{\gamma} = \det \hat{\gamma}_{\hat{i}\hat{j}}$, and the target space is a supermanifold with the superspace coordinates $Z^{\hat{M}} = (X^M, \theta^\alpha)$ $(M = 0, \cdots, 10,\ \alpha = 1, \cdots, 32)$. Furthermore, with the tangent superspace index $\hat{A} = (A, a)$, $\hat{E}_{\hat{M}}^{\hat{A}}$ is the supervielbein and η_{AB} is the tangent space metric in eleven dimensions. The mass dimension of the worldvolume parameter $\sigma^{\hat{i}}$ and eleven-dimensional background fields (G_{MN}, $\hat{C}_{MNP}$) is 0, while that of the worldvolume metric $\hat{\gamma}_{\hat{i}\hat{j}}$ is -2.

We shall investigate the origin of the $R^{(2)}\phi$-term in the dimensionally reduced string theory. In string theory, the anomaly of the reparametrization ghost number current gives $\sqrt{-\gamma}\, R^{(2)}$, which is (a local version of) the Riemann-Roch theorem, and hence, similarly to Ref.[9], we may focus on the bosonic degrees of freedom to investigate the $R^{(2)}\phi$-term from the supermembrane theory.

[1] The eleven-dimensional Planck length l_{11} is defined by $T = (2\pi)^{-2} l_{11}^{-3}$.

The bosonic background fields are included in the superfields as

$$\hat{E}_M^{\ A}(Z)\Big|_{\text{fermions}=0} = \hat{e}_M^{\ A}(X)\,, \quad \hat{C}_{MNP}(Z)\Big|_{\text{fermions}=0} = \hat{A}_{MNP}(X)\,. \tag{2.3}$$

Then, the action (2.1) is reduced to

$$\begin{aligned} S \;=\; T\int d^3\sigma\Big[& -\frac{1}{2}\sqrt{-\hat{\gamma}}\,\hat{\gamma}^{\hat{i}\hat{j}}\,\partial_{\hat{i}}X^M\partial_{\hat{j}}X^N G_{MN}(X) + \frac{1}{2}\sqrt{-\hat{\gamma}} \\ & -\frac{1}{3!}\,\epsilon^{\hat{i}\hat{j}\hat{k}}\,\partial_{\hat{i}}X^M\partial_{\hat{j}}X^N\partial_{\hat{k}}X^P\hat{A}_{PNM}(X)\Big]. \end{aligned} \tag{2.4}$$

Note that the variation w.r.t. $\hat{\gamma}_{\hat{i}\hat{j}}$ yields the induced metric,

$$\hat{\gamma}_{\hat{i}\hat{j}} = \partial_{\hat{i}}X^M\partial_{\hat{j}}X^N G_{MN}(X) \equiv G_{\hat{i}\hat{j}}\,, \tag{2.5}$$

and plugging it back into the original action leads to the one in the Nambu-Goto form

$$S^{(\mathrm{NG})} = T\int d^3\sigma\left[-\sqrt{-\det G_{\hat{i}\hat{j}}} - \frac{1}{3!}\,\epsilon^{\hat{i}\hat{j}\hat{k}}\partial_{\hat{i}}X^M\partial_{\hat{j}}X^N\partial_{\hat{k}}X^P\hat{A}_{PNM}(X)\right]. \tag{2.6}$$

In Ref.[9], they examined the membrane partition function

$$Z = \sum_{\text{topologies}}\int\frac{\mathcal{D}X\mathcal{D}\hat{\gamma}}{\mathrm{Vol}(\mathrm{Diff}_0)}\,e^{-S}\,, \tag{2.7}$$

under the assumption that the membrane was wrapped once around the S^1-compactified 11th direction of the target space and they truncated to the zero-mode sector of the circle. That is, the worldvolume topology was assumed to be $S^1\times\Sigma$ with Σ being some Riemann surface and the target space topology was $M^{10}\times S^1$ with S^1 being the 11th direction of M-theory or the M-theory circle, and the double dimensional reduction [2] was applied. They fixed some variables

$$\sigma^2 = X^{11}\,, \quad \hat{\gamma}_{22}=1\,, \quad \hat{\gamma}_{02}=\hat{\gamma}_{12}=0\,, \quad G_{1010}=1\,, \tag{2.8}$$

and analyzed the path integral measure of the partition function (2.7). They found that the norm of the variation of the worldvolume metric leads to the relation

$$||\delta\hat{\gamma}_{ij}|| = \sqrt{R_{11}}\,||\delta\gamma_{ij}||, \tag{2.9}$$

where R_{11} is the radius of the M-theory circle. Then, (2.9) is followed by the relation between the moduli space (or the conformal Killing vectors) measures of the string and the dimensionally reduced wrapped membrane, which leads to the $\chi\phi$-term to the string action.

In this paper, we reinvestigate the partition function (2.7). Similarly to Ref.[9], since we are interested in the coupling between the dilaton and the worldsheet curvature in the reduced string theory, we consider the S^1-compactified wrapped supermembrane, for simplicity. We take the X^{10}-directions to be compactified on S^1 of radius L_1 and hence the worldvolume of the membrane is at least locally $\Sigma_{ws}\times S^1$, where Σ_{ws} is a Riemann surface and S^1 is to be parametrized by σ^2. Then, we shall represent the wrapping of the supermembrane as

$$\sqrt{\mathring{G}_{1010}}\,X^{10}(\sigma^{\hat{i}}+2\pi\delta_{\hat{i}2}) = 2\pi w_1 L_1 + \sqrt{\mathring{G}_{1010}}\,X^{10}(\sigma^{\hat{i}})\,, \tag{2.10}$$

where $\mathring{G}_{1010}$ stands for the asymptotic values of the corresponding component of the target space metric and $w_1 \in \mathbb{N}$ is a wrapping number. For simplicity, we put $w_1 = 1$ hereafter. From eq.(A.1) we shall see

$$R_1 \equiv \frac{L_1}{\sqrt{\mathring{G}_{1010}}} = L_1\, e^{-2\phi_0/3}, \tag{2.11}$$

where ϕ_0 is the asymptotic constant value of the type IIA dilaton background and hence the M/IIA-relation or 11d/IIA-SUGRA-relation leads to

$$R_1 = l_{11}\,. \tag{2.12}$$

We parametrize the worldvolume metric $\hat{\gamma}_{\hat{i}\hat{j}}$ as

$$\left(\hat{\gamma}_{\hat{i}\hat{j}}\right) = l_{11}^2 \begin{pmatrix} h^{-1/2}\,\gamma_{ij} + hV_iV_j & h\,V_i \\ h\,V_j & h \end{pmatrix}, \tag{2.13}$$

where γ_{ij}, h and V_i are dimensionless, and we have[2]

$$\begin{aligned} \det(\hat{\gamma}_{\hat{i}\hat{j}}) &= l_{11}^6 \det(\gamma_{ij})\,, & (2.16)\\ \left(\hat{\gamma}_{\hat{i}\hat{j}}\right)^{-1} &= \left(\hat{\gamma}^{\hat{i}\hat{j}}\right) = l_{11}^{-2} \begin{pmatrix} h^{1/2}\,\gamma^{ij} & -h^{1/2}\,\gamma^{ik}V_k \\ -h^{1/2}\,\gamma^{jk}V_k & h^{-1} + h^{1/2}V_k\gamma^{kl}V_l \end{pmatrix}. & (2.17) \end{aligned}$$

Then, the action (2.4) is rewritten as

$$\begin{aligned} S \;=\; \frac{T}{2}\int d^3\sigma \Big[& -l_{11}\sqrt{-\gamma}\,h^{1/2}\gamma^{ij}\,(\partial_i X\cdot\partial_j X - l_{11}^2\hat{h}\hat{V}_i\hat{V}_j) - l_{11}^3\sqrt{-\gamma}\,(h^{-1}\hat{h} - 1) \\ & - l_{11}^3\sqrt{-\gamma}\,h^{1/2}\hat{h}\,\gamma^{ij}(V_i - \hat{V}_i)(V_j - \hat{V}_j) + \epsilon^{ij}\,\partial_i X^M\partial_j X^N\partial_2 X^P \hat{A}_{MNP}\Big], \end{aligned} \tag{2.18}$$

where

$$\partial_{\hat{i}} X\cdot\partial_{\hat{j}} X = \partial_{\hat{i}} X^M\partial_{\hat{j}} X^N G_{MN}\,,\quad \hat{h} = l_{11}^{-2}\,\partial_2 X\cdot\partial_2 X\,,\quad \hat{V}_i = \frac{\partial_i X\cdot\partial_2 X}{\partial_2 X\cdot\partial_2 X}\,. \tag{2.19}$$

We shall make a gauge choice for the worldvolume diffeomorphism.[3] We adopt the

[2]Note that for an arbitrary worldvolume vector $\hat{A}_{\hat{i}}$ and a tensor $\hat{A}_{\hat{i}\hat{j}}$, we have

$$\begin{aligned} &\hat{A}^{\hat{i}} = \hat{\gamma}^{\hat{i}\hat{k}}\hat{A}_{\hat{k}} \to \hat{A}^i = l_{11}^{-2}h^{1/2}\gamma^{ik}(\hat{A}_k - V_k\hat{A}_2)\,, \\ &\hat{A}^{\hat{i}\hat{j}} = \hat{\gamma}^{\hat{i}\hat{k}}\hat{\gamma}^{\hat{j}\hat{l}}\hat{A}_{\hat{k}\hat{l}} \to \hat{A}^{ij} = l_{11}^{-4}h\gamma^{ik}\gamma^{jl}(\hat{A}_{kl} - V_k\hat{A}_{2l} - \hat{A}_{k2}V_l + V_kV_l\hat{A}_{22})\,. \end{aligned} \tag{2.14}$$

Meanwhile, the first two components ϵ^i of a parameter $\epsilon^{\hat{i}}$ for the worldvolume diffeomorphism can be a parameter of the worldsheet diffeomorphism because, for example, $(\mu = 0, 1, \cdots, 9)$

$$\delta X^M = \epsilon^{\hat{i}}\partial_{\hat{i}} X^M \underset{DDR}{\to} \delta X^\mu = \epsilon^i\partial_i X^\mu. \tag{2.15}$$

[3]Under the diffeomorphism, the components in (2.13) are transformed as

$$\begin{aligned} \delta\gamma_{ij} &= \epsilon^{\hat{i}}\partial_{\hat{i}}\gamma_{ij} + \gamma_{ij}\partial_2\epsilon^2 + (\gamma_{ij}V_k - \gamma_{jk}V_i - \gamma_{ki}V_j)\partial_2\epsilon^k + \gamma_{ik}\partial_j\epsilon^k + \gamma_{jk}\partial_i\epsilon^k, & (2.20)\\ \delta V_i &= \epsilon^{\hat{i}}\partial_{\hat{i}} V_i + V_j\,\partial_i\epsilon^j + (h^{-3/2}\gamma_{ij} - V_iV_j)\,\partial_2\epsilon^j + \partial_i\epsilon^2 - V_i\partial_2\epsilon^2, & (2.21)\\ \delta h &= \epsilon^{\hat{i}}\partial_{\hat{i}} h + 2h\,\partial_2\epsilon^2 + 2hV_i\,\partial_2\epsilon^i. & (2.22) \end{aligned}$$

following gauge conditions[4]

$$X^{10} = R_1\,\sigma^2, \quad \gamma_{01} = \gamma_{00} + \gamma_{11} = 0\,. \tag{2.23}$$

Under the gauge condition (2.23) the action (2.4) or (2.18) becomes

$$\begin{aligned} S_{\rm Gfd} \;=\; & \frac{T}{2}\int d^3\sigma\,\Big[-\frac{h^{1/2}\eta^{ij}}{G_{1010}^{1/2}}\Big\{l_{11}\partial_i X^\mu \partial_j X^\nu g_{\mu\nu} - 2\frac{G_{1010}}{\hat h}\,\partial_i X^\mu \partial_j X^\nu \partial_2 X^\rho g_{\mu\rho} A_\nu \\ & + \frac{G_{1010}}{l_{11}\hat h}\,\partial_i X^\mu \partial_j X^\nu \partial_2 X^\rho \partial_2 X^\sigma \Big(A_\mu A_\nu g_{\rho\sigma} - 2A_\nu A_\sigma g_{\mu\rho} - \frac{g_{\mu\rho} g_{\nu\sigma}}{G_{1010}^{3/2}}\Big)\Big\} \\ & + l_{11}\epsilon^{ij}\partial_i X^\mu \partial_j X^\nu B_{\mu\nu} + \epsilon^{ij}\partial_i X^\mu \partial_j X^\nu \partial_2 X^\rho A_{\mu\nu\rho} \\ & - l_{11}^3 \rho h^{-1}(\hat h - h) - l_{11}^3 h^{1/2}\hat h\,\eta^{ij}(V_i - \hat V_i)(V_j - \hat V_j)\Big], \end{aligned} \tag{2.24}$$

where γ_{ij} has been fixed to be the fiducial metric $\bar\gamma_{ij}$ by the gauge condition (2.23)

$$\bar\gamma_{ij} \equiv \rho\eta_{ij}, \tag{2.25}$$

and use has been made of the (10+1)-decomposition of the background fields in (A.1) and (A.2).

The FP-determinant $\Delta_{\rm FP}$, or the Fadeev-Popov ghost action $S_{\rm FP}$, associated with the gauge condition (2.23) is given by (see appendix B and also [15])

$$\Delta_{\rm FP}(\hat\gamma, X) \;=\; \int \mathcal{D}\hat b_{++}\,\mathcal{D}\hat b_{--}\,\mathcal{D}\hat c^{+}\,\mathcal{D}\hat c^{-}\,\exp\Big(iS_{\rm FP}\Big), \tag{2.26}$$

$$S_{\rm FP} \;=\; -\frac{i}{2\pi^2}\int d^3\sigma\, h^{1/2}\Big\{\hat b_{--}(\partial_+ - V_+\partial_2)\hat c^- + \hat b_{++}(\partial_- - V_-\partial_2)\hat c^+\Big\}, \tag{2.27}$$

where $\hat b_{\pm\pm}$ and $\hat c^{\pm}$ are Grassmann odd ghosts with ghost numbers -1 and $+1$, respectively, and

$$\partial_\pm = \frac{1}{2}\,(\partial_0 \pm \partial_1), \quad V_\pm = \frac{1}{2}\,(V_0 \pm V_1)\,. \tag{2.28}$$

The total action is given by

$$S_T = S_{\rm Gfd} + S_{\rm FP}. \tag{2.29}$$

Now that we make the double dimensional reduction by imposing the conditions to deduce the type IIA string ($\mu = 0, 1, \cdots, 9$)

$$\partial_2 X^\mu\,(=\partial_{\sigma^2} X^\mu) = 0\,, \quad \frac{\partial}{\partial X^{10}}\,G_{MN} = \frac{\partial}{\partial X^{10}}\,\hat A_{MNP} = 0\,. \tag{2.30}$$

By the double dimensional reduction the total action is reduced to

$$\begin{aligned} S_T \underset{DDR}{\longrightarrow} S_{st} \;=\; & \frac{T_s}{2}\int d^2\sigma\,\Big[-\sqrt{\frac{h}{\bar h}}\,\eta^{ij}\partial_i X^\mu \partial_j X^\nu g_{\mu\nu} + \epsilon^{ij}\partial_i X^\mu \partial_j X^\nu B_{\mu\nu} \\ & - \frac{1}{2\pi T_s}\rho\Big(\frac{\bar h}{h} - 1\Big) - \frac{1}{2\pi T_s} h^{1/2}\bar h\,\eta^{ij}(V_i - \bar V_i)(V_j - \bar V_j) \\ & - \frac{2i}{\pi T_s} h^{1/2}(b_{--}\partial_+ c^- + b_{++}\partial_- c^+)\Big], \end{aligned} \tag{2.31}$$

[4]We may adopt $\partial_2 X^{10} = R_1$ instead of $X^{10} = R_1\sigma^2$ to see the conformal transformation explicitly in the dimensionally reduced string theory;[14] however, it does not make an essential difference in our analysis.

where

$$T_s = 2\pi R_1 T = (2\pi l_{11}^2)^{-1}, \tag{2.32}$$

$$\hat{h} \underset{DDR}{\rightarrow} G_{1010} \equiv \bar{h}\,, \quad \hat{V}_i \underset{DDR}{\rightarrow} \frac{G_{\mu 10}\partial_i X^\mu}{R_1 G_{1010}} \equiv \bar{V}_i\,, \tag{2.33}$$

and the variables in S_{st} are understood to be independent of σ^2. Note that V_i can be integrated out straightforwardly, while the field equation for ρ is

$$h = \bar{h}\,. \tag{2.34}$$

As was pointed out in [14], substituting this algebraic condition back into S_{st} yields a ρ-independent action. However, ρ is not free but is related, for example, to X^μ through the equation of motion for h. The Euclidean action corresponding to S_{st} is

$$\begin{aligned} S_{st}^E &= T_s \int d^2 z \Bigg[\sqrt{\frac{h}{\bar{h}}}\, \partial_z X^\mu \partial_{\bar{z}} X^\nu g_{\mu\nu} + \partial_z X^\mu \partial_{\bar{z}} X^\nu B_{\mu\nu} + \frac{1}{8\pi T_s} \rho \left(\frac{\bar{h}}{h} - 1 \right) \\ &\quad + \frac{1}{2\pi T_s} h^{1/2} \bar{h}\, (V_z - \bar{V}_z)(V_{\bar{z}} - \bar{V}_{\bar{z}}) + \frac{1}{2\pi T_s}\, h^{1/2} (b_{zz} \partial_{\bar{z}} c^z + b_{\bar{z}\bar{z}} \partial_z c^{\bar{z}}) \Bigg]. \end{aligned} \tag{2.35}$$

3 Dilaton coupling

In this section we examine the partition function with the double dimensional reduced membrane action. First, we shall make the BRS invariant path integral measure according to the Fujikawa method [10]. In this case, the integration variables for the path integral are obtained as follows: For a worldvolume scalar X^μ we have

$$\begin{aligned} &\int \mathcal{D}\mathring{X}^M \exp\left[T l_{11}^{-2} \int d^3\sigma \sqrt{-\hat{\gamma}}\, X^M X^N G_{MN} \right] \\ &= \int \mathcal{D}\mathring{X}^M \exp\left[T l_{11} \int d^3\sigma \, \sqrt{-\gamma} \left\{ X^\mu X^\nu \frac{g_{\mu\nu}}{\sqrt{G_{1010}}} + G_{1010} \Big(X^{10} + \frac{G_{10\mu} X^\mu}{G_{1010}} \Big)^2 \right\} \right]. \end{aligned} \tag{3.1}$$

This implies (we shall set $\det g_{\mu\nu} = -1$)

$$\mathcal{D}\mathring{X}^\mu = \mathcal{D}\Big((-\gamma/\bar{h})^{1/4} X^\mu \Big). \tag{3.2}$$

Meanwhile, we have the following decomposition for a worldvolume vector,

$$l_{11}^{-2} \hat{\gamma}_{\hat{i}\hat{j}} A^{\hat{i}} A^{\hat{j}} = h^{-1/2} \gamma_{ij} A^i A^j + h (A^2 + A^i V_i)^2\,. \tag{3.3}$$

This implies ($\gamma_{ij} = \tilde{e}_i{}^a \tilde{e}_{ja}$, ($\tilde{e}_i{}^a$: zweibein))

$$\mathcal{D}\mathring{A}^{\hat{a}} = \mathcal{D}\Big((-\gamma/h)^{1/4} \tilde{e}_i{}^a A^i \Big)\, \mathcal{D}\Big((-\gamma h^2)^{1/4} A^2 \Big) = \mathcal{D}\Big((-\gamma)^{1/2} h^{-1/4} A^i \Big)\, \mathcal{D}\Big((-\gamma)^{1/4} h^{1/2} A^2 \Big)\,. \tag{3.4}$$

Similarly, for a symmetric tensor we have

$$\mathcal{D}\mathring{\Phi}_{\hat{a}\hat{b}} = \mathcal{D}\Big((-\gamma)^{-1/4} h^{1/2} \Phi_{ij} \Big) \mathcal{D}\Big(h^{-1/4} \Phi_{2i} \Big) \mathcal{D}\Big((-\gamma)^{1/4} h^{-1} \Phi_{22} \Big)\,. \tag{3.5}$$

Let us rewrite S_{st} with the path integral variables $\mathring{X}, \mathring{b}$ and $\mathring{c}$. We have (cf. (3.2), (3.4), (3.5))

$$\begin{aligned} \mathring{X}^\mu &= \rho^{1/2}\,\bar{h}^{-1/4} X^\mu, \\ \mathring{c}^z &= \rho\, h^{-1/4} c^z, \\ \mathring{b}_{zz} &= \rho^{-1/2} h^{1/2} b_{zz}\,. \end{aligned} \tag{3.6}$$

Thus, S^E_{st} becomes

$$\begin{aligned} S^E_{st} &= \int d^2z \Big[T_s \sqrt{\frac{h}{\bar{h}}}\, \partial_z\Big(\frac{\mathring{X}^\mu}{\rho^{1/2}\bar{h}^{-1/4}}\Big)\partial_{\bar{z}}\Big(\frac{\mathring{X}^\nu}{\rho^{1/2}\bar{h}^{-1/4}}\Big) g_{\mu\nu} \\ &\quad + T_s\sqrt{\frac{h}{\bar{h}}}\, \partial_z\Big(\frac{\mathring{X}^\mu}{\rho^{1/2}\bar{h}^{-1/4}}\Big)\partial_{\bar{z}}\Big(\frac{\mathring{X}^\nu}{\rho^{1/2}\bar{h}^{-1/4}}\Big) B_{\mu\nu} + \frac{1}{8\pi}\rho\Big(\frac{\bar{h}}{h} - 1\Big) \\ &\quad + \frac{1}{2\pi}\,\rho^{1/2}\mathring{b}_{zz}\partial_{\bar{z}}\Big(\rho^{-1}h^{1/4}\mathring{c}^z\Big) + \frac{1}{2\pi}\,\rho^{1/2}\mathring{b}_{\bar{z}\bar{z}}\partial_z\Big(\rho^{-1}h^{1/4}\mathring{c}^{\bar{z}}\Big)\Big], \end{aligned} \tag{3.7}$$

and the partition function is

$$Z \sim \int \mathcal{D}\mathring{X}^\mu \mathcal{D}\mathring{c}^z \mathcal{D}\mathring{c}^{\bar{z}} \mathcal{D}\mathring{b}_{zz} \mathcal{D}\mathring{b}_{\bar{z}\bar{z}}\, e^{-S^E_{st}}. \tag{3.8}$$

We find that ρ and also h can be removed from the action S^E_{st} by putting $h = \bar{h}$ and making the following rescalings,

$$(\mathring{X}, \mathring{b}, \mathring{c}) \;\to\; (e^{\alpha_c/2}\mathring{X}, e^{-\alpha_c/2}\mathring{b}, e^{\alpha_c}\mathring{c}), \quad (\, e^{\alpha_c} = \rho\bar{h}^{-1/2}\,) \tag{3.9}$$

$$(\mathring{X}, \mathring{b}, \mathring{c}) \;\to\; (\mathring{X}, e^{-\alpha_g}\mathring{b}, e^{\alpha_g}\mathring{c}). \qquad (\, e^{\alpha_g} = (\bar{h}^2/h)^{1/4}\,) \tag{3.10}$$

However, the rescalings in (3.9) and (3.10) shall generate the jacobians from the path integral measure [10]. We can see that (3.9) is a conformal transformation and (3.10) is a ghost number transformation for the worldsheet reparametrization ghosts. Since the superstring induced from the wrapped supermembrane is in the critical dimension, the jacobian coming from (3.9) should be trivial, or the conformal anomaly should be canceled.[5] On the other hand, the jacobian from (3.10) corresponds to the reparametrization ghost number anomaly and it is well known that the anomaly equation gives the local version of the Riemann-Roch theorem,

$$\partial_{\bar{z}} j^{gh}_z = -\frac{3}{4\pi}\partial_z\partial_{\bar{z}}\ln\rho = \frac{3}{8\pi}\sqrt{\gamma^E}R^{(2)}, \tag{3.11}$$

where j^{gh}_z is the worldsheet reparametrization ghost number current and $R^{(2)}$ is the worldsheet curvature. Thus, we have

$$\begin{aligned} \mathcal{D}(e^{-\alpha_g}\mathring{b})\mathcal{D}(e^{\alpha_g}\mathring{c}) &= \mathcal{D}\mathring{b}\mathcal{D}\mathring{c}\, J(\alpha_g) = \mathcal{D}\mathring{b}\mathcal{D}\mathring{c}\, e^{\ln J} \\ &= \mathcal{D}\mathring{b}\mathcal{D}\mathring{c}\, \exp\Big[-\frac{3}{8\pi}\int d^2z\, \Phi(\alpha_g)\sqrt{\gamma^E}\, R^{(2)}\Big], \end{aligned} \tag{3.12}$$

where

$$\Phi(\alpha_g) = \ln\alpha_g = \frac{1}{4}\ln\left(\frac{\bar{h}^2}{h}\right) \simeq \frac{1}{4}\ln G_{1010} = \frac{1}{3}\,\phi\,, \tag{3.13}$$

[5] Of course, we should recover the fermionic coordinates and the bosonic ghosts for the supertransformation in order to cancel out the conformal anomaly (see section 4).

and ϕ is the background dilaton (cf. (A.1)). Then, we have ($^\circ$ is omitted)

$$\mathcal{D}(e^{-\alpha_g}b_{zz})\mathcal{D}(e^{-\alpha_g}b_{\bar{z}\bar{z}})\mathcal{D}(e^{\alpha_g}c^z)\mathcal{D}(e^{\alpha_g}c^{\bar{z}}) = \mathcal{D}b_{zz}\mathcal{D}b_{\bar{z}\bar{z}}\mathcal{D}c^z\mathcal{D}c^{\bar{z}}\ \exp\Big[-\frac{1}{4\pi}\int d^2z\sqrt{\gamma^E}\,\phi R^{(2)}\Big]. \tag{3.14}$$

We have seen that this dilaton-curvature term has appeared due to h, which indicates that the term originated with the membrane.

4 Discussion

In this paper, we have studied the dilaton coupling in the string theory, which is induced from a wrapped membrane. We have seen that the dilaton coupling to the string worldsheet curvature comes out from the path integral measure due to the ghost number anomaly, which originates from the fact that the string is reduced from the membrane.

There remains some points to be discussed here. The induced string action (2.31) or (2.35) has the conformal mode ρ dependence. This action is obtained from the Polyakov-type action (2.4) through the double dimensional reduction. On the other hand, once we start with the Nambu-Goto action (2.6) and adopt essentially the same gauge conditions as (2.23),

$$X^{10} = R_1\sigma^2, \quad \gamma^X_{01} = \gamma^X_{00} + \gamma^X_{11} = 0, \tag{4.1}$$

where γ^X_{ij} is the induced metric (cf. (2.5), (2.13))

$$\gamma^X_{ij} = l_{11}^{-1}G_{22}^{1/2}G_{ij} - l_{11}^{-2}G_{22}^{-1}G_{i2}G_{j2}\,, \tag{4.2}$$

the total action in this case becomes

$$\begin{aligned} S_T^{(\mathrm{NG})} \quad = \quad & \frac{Tl_{11}^3}{2}\int d^3\sigma\,\Big[-(\gamma^X_{00}-\gamma^X_{11}) - l_{11}^{-2}\epsilon^{ij}\partial_iX^\mu\partial_jX^\nu B_{\mu\nu} - l_{11}^{-3}\epsilon^{ij}\partial_iX^\mu\partial_jX^\nu\partial_2X^\rho A_{\mu\nu\rho} \\ & - 4i\hat{h}^{1/2}\Big\{\hat{b}_{++}(\partial_- - \hat{V}_-\partial_2)\hat{c}^+ + \hat{b}_{--}(\partial_+ - \hat{V}_+\partial_2)\hat{c}^-\Big\}\Big]\,. \end{aligned} \tag{4.3}$$

We shall see that $S_T^{(\mathrm{NG})}$ corresponds to S_T (2.29) with $h = \hat{h}$ and $V_i = \hat{V}_i$. Thus, the dilaton-curvature coupling term (3.14) will be followed by a similar analysis.

In this paper we have omitted the fermionic coordinates and hence the worldvolume local supersymmetry. However, we have used the fact that the jacobian coming from (3.9) is trivial, which is to be explicitly shown with the fermionic coordinates. Since the wrapped supermembrane is reduced to the Green-Schwarz type of superstring, but not the Neveu-Schwarz-Ramond type, such a gauge fixing as in [16] will be desired. This should be studied further.

We have considered the S^1-compactification of the supermembrane. In the meantime, it was shown explicitly that the wrapped supermembrane on a 2-torus induces the (p,q)-string [7, 5]. Then, the dilaton-curvature coupling term in the type IIB superstring would be directly presented with the wrapped supermembrane.

We adopted the double dimensional reduction for the wrapped supermembrane and did not investigate the Kaluza-Klein modes. Meanwhile, quantum mechanical justifications of the double dimensional reduction were studied in [17, 18]. In order to study the coupling between ρ and the Kaluza-Klein modes, such quantum mechanical treatment should be investigated further.

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A 11d vs. 10d background fields

The 11-dimensional metric can be written as

$$\begin{aligned} G_{MN} &\equiv e^{-\frac{2}{3}\phi}\begin{pmatrix} g_{\mu\nu}+e^{2\phi}A_\mu A_\nu & e^{2\phi}A_\mu \\ e^{2\phi}A_\nu & e^{2\phi}\end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{\sqrt{G_{1010}}}\,g_{\mu\nu}+\frac{1}{G_{1010}}G_{\mu 10}G_{\nu 10} & G_{\mu 10} \\ G_{\nu 10} & G_{1010}\end{pmatrix}, \end{aligned} \tag{A.1}$$

and the third-rank antisymmetric tensor $\hat{A}_{MNP}$ is decomposed as

$$\{\hat{A}_{MNP}\} = \{\hat{A}_{\mu\nu\rho}, \hat{A}_{\mu\nu 10}\} = \{A_{\mu\nu\rho}, B_{\mu\nu}\}\,. \tag{A.2}$$

B FP determinant (2.27)

Let us calculate the FP determinant. The gauge condition (2.23)

$$X^{10} = R_1\,\sigma^2, \quad \gamma_{01} = \gamma_{00}+\gamma_{11} = 0, \tag{B.1}$$

leads to (cf. (2.13))

$$\begin{aligned} \Delta_{\rm FP}(\hat{\gamma}, X)^{-1} &= \int \mathcal{D}\hat{\zeta}\,\delta((X^{10})^{\hat{\zeta}} - R_1\,\sigma^2)\,\delta((\hat{\gamma}_{\hat{i}\hat{j}} - \hat{\bar{\gamma}}_{\hat{i}\hat{j}})^{\hat{\zeta}}) \\ &= \int \mathcal{D}\hat{\zeta}\,\delta(R_1\hat{\zeta}^2)\,\delta(h^{-1/2}\delta_{\hat{\zeta}}(\Delta\gamma_{ij})) \\ &= \int \mathcal{D}\hat{\zeta}\,\delta(R_1\hat{\zeta}^2)\int \mathcal{D}\hat{\lambda}\,\exp\Big[2\pi i\int\sqrt{-\hat{\gamma}}\,h^{-1/2}\hat{\lambda}^{ij}\delta_{\hat{\zeta}}(\Delta\gamma_{ij})\Big] \\ &= \int \mathcal{D}\hat{\zeta}\,\delta(R_1\hat{\zeta}^2)\int \mathcal{D}\hat{\lambda}\,\exp\Big[2\pi i\int\sqrt{-\bar{\gamma}}\,h^{1/2}\,\check{\lambda}^{ij}\delta_{\hat{\zeta}}(\Delta\gamma_{ij})\Big], \end{aligned} \tag{B.2}$$

where $\hat{\bar{\gamma}}_{\hat{i}\hat{j}}$ is given by substituting the fiducial metric $\bar{\gamma}_{ij}$ (2.25) for γ_{ij} in $\hat{\gamma}_{\hat{i}\hat{j}}$, $\Delta\gamma_{ij}$ is the difference between γ_{ij} and $\bar{\gamma}_{ij}$,

$$\Delta\gamma_{ij} = \gamma_{ij} - \bar{\gamma}_{ij}, \tag{B.3}$$

$\check{\lambda}^{ij}$ is traceless, $\check{\lambda}^{ij}\bar{\gamma}_{ij} = 0$, and $\delta_{\hat{\zeta}}(\Delta\gamma_{ij}) = \delta_{\hat{\zeta}}(\gamma_{ij} - \bar{\gamma}_{ij})|_{\gamma_{ij}=\bar{\gamma}_{ij}}$. Thus, eq.(2.27) is followed by

$$\begin{aligned} \check{\lambda}^{ij}\delta_{\hat{\zeta}}(\Delta\gamma_{ij}) &= 2\check{\lambda}^{01}\bar{\gamma}_{00}(V_0\partial_2\hat{\zeta}^1 - V_1\partial_2\hat{\zeta}^0 + \partial_1\hat{\zeta}^0 - \partial_0\hat{\zeta}^1) \\ &\quad + 2\check{\lambda}^{00}\bar{\gamma}_{00}(V_1\partial_2\hat{\zeta}^1 - V_0\partial_2\hat{\zeta}^0 + \partial_0\hat{\zeta}^0 - \partial_1\hat{\zeta}^1) \\ &= 2\bar{\gamma}^{11}\check{\lambda}_{01}(V_0\partial_2\hat{\zeta}^1 - V_1\partial_2\hat{\zeta}^0 + \partial_1\hat{\zeta}^0 - \partial_0\hat{\zeta}^1) \\ &\quad + 2\bar{\gamma}^{00}\check{\lambda}_{00}(V_1\partial_2\hat{\zeta}^1 - V_0\partial_2\hat{\zeta}^0 + \partial_0\hat{\zeta}^0 - \partial_1\hat{\zeta}^1) \\ &= \frac{4}{\sqrt{-\det\bar{\gamma}}}\Big\{\check{\lambda}_{++}(V_-\partial_2 - \partial_-)\hat{\zeta}^+ + \check{\lambda}_{--}(V_+\partial_2 - \partial_+)\hat{\zeta}^-\Big\}, \end{aligned} \tag{B.4}$$

where

$$\hat{\zeta}^\pm = \hat{\zeta}^0 \pm \hat{\zeta}^1, \quad \partial_\pm = \frac{1}{2}(\partial_0 \pm \partial_1), \quad V_\pm = \frac{1}{2}(V_0 \pm V_1), \quad \check{\lambda}_{\pm\pm} = \frac{1}{2}(\check{\lambda}_{00} \pm \check{\lambda}_{01}). \tag{B.5}$$

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